INTRODUCTION TO GEOMETRIC GROUP THEORY

Igor Mineyev. Math 503, Spring 2026. MWF 3pm. mineyev.web.illinois.edu/class/26s/503/
Note: **Math 500 will NOT be required as a prerequisite** for this course; disregard what Course Explorer says. Familiarity with standard notions in group theory is helpful.
Register for this class early to make sure this 503 course will actually run. And feel free to advertise.

Geometric group theory is not a subject in itself; it is rather the place where various areas of mathematics interact: algebra, topology, geometry, analysis, computational methods, and more. Here is the tentative list of topics that I intend to cover in this course; this might be modified somewhat as we proceed.

- Cayley graphs, the word metric, groups as metric spaces, quasiisometry.
- One-dimensional things: Free groups and their subgroups, their descriptions via Stallings' graphs, the Nielsen-Schreier subgroup theorem, Nielsen transformations, automorphisms of free groups.
- Group actions on trees, free products, ping-pong lemma, free products with amalgamations, HNN-extensions, graphs of groups.
- Two-dimensional things: Groups presentations by generators and relators, van Kampen diagrams, van Kampen theorem, isoperimetric function, algorithmic problems in group theory.
- Examples of quasiisometry invariants: growth of finitely generated groups, ends, isoperimetric functions, amenability, solvability of the word problem, asymptotic cones, hyperbolicity.
- Multi-dimensional things: Word hyperbolic groups and spaces, their numerous definitions and properties, examples, the ideal boundary, quasiconformal and conformal structures on the ideal boundary, cubical complexes, . . .

No specific textbook is required for the course. Check the following additional sources.

- If interested, *Links and comments on geometric group theory* are available from *my Math Page.* That page contains a list of sources one can use to learn more about various facets of geometric group theory, and comments on how it is related to other subjects.
- The following online games are related to geometric group theory. Play the games and click on their titles to see more information.
  - ColorTaiko! game: play.math.illinois.edu/ColorTaiko!/
  - PathForms game: play.math.illinois.edu/PathForms/
- My Math 490 course *Constructive geometry: from grounds to groups* in Fall 2025 gives a highly unconventional, constructive-geometric introduction to group theory using *grounds*.