ALGEBRAIC TOPOLOGY I

Igor Mineyev. Math 525, Spring 2024. MWF 2:00-2:50 p.m., 329 Gregory Hall.
Textbook: *Algebraic topology* by Allen Hatcher. Freely available online at
`https://pi.math.cornell.edu/~hatcher/AT/ATpage.html`

The tentative syllabus. This is the official syllabus, we will roughly follow it in the course.
Fundamental group and covering spaces.

(1) Definition of the fundamental group.
(2) Covering spaces and lifts of maps.
(3) Computing the fundamental group via covering spaces.
(4) Applications, such as the Fundamental Theorem of Algebra and the Brouwer fixed point theorem in 2d.
(5) Deforming spaces: retraction and homotopy equivalence.
(6) Quotient topology and cell complexes.
(7) Homotopy extension property and applications to homotopy equivalence.
(8) Fundamental groups of CW complexes.
(9) Van Kampen's Theorem.
(10) Covering spaces and subgroups of the fundamental group.
(11) Universal covers.
(12) The definitive lifting criterion, classification of covering spaces.
(13) Covering transformations and regular covers.

Homology.

(14) Delta complexes and their cellular homology.
(15) Singular homology.
(16) Homotopic maps and homology.
(17) The long exact sequence of the pair.
(18) Relative homology and excision.
(19) Equality of cellular and singular homology.
(20) Applications, such as degree of maps of spheres, invariance of dimension, and the Brouwer fixed point theorem.
(21) Homology of CW complexes.
(22) Homology and the fundamental group: the Hurewicz theorem.
(23) Euler characteristic.
(24) Homology with coefficients.
(25) Intro to categories and axiomatic characterization of homology theories.
(26) Further applications, such as the Jordan curve theorem, wild spheres, invariance of domain.