INTRODUCTION TO GEOMETRIC GROUP THEORY

Igor Mineyev. Math 503, Spring 2024. MWF 3pm.

Geometric group theory is not a subject in itself; it is rather the place where various areas of mathematics interact: algebra, topology, geometry, analysis, computational methods, and more. Here is the tentative list of topics that I intend to cover in this course; this might be modified somewhat as we proceed.

- Cayley graphs, the word metric, groups as metric spaces, quasiisometry.
- One-dimensional things: Free groups and their subgroups, their descriptions via Stallings' graphs, the Nielsen-Schreier subgroup theorem, Nielsen transformations, automorphisms of free groups.
- Group actions on trees, free products, ping-pong lemma, free products with amalgamations, HNN-extensions, graphs of groups.
- Two-dimensional things: Groups presentations by generators and relators, van Kampen diagrams, van Kampen theorem, isoperimetric function, algorithmic problems in group theory.
- Examples of quasiisometry invariants: growth of finitely generated groups, ends, isoperimetric functions, amenability, solvability of the word problem, asymptotic cones, hyperbolicity.
- Multi-dimensional things: Word hyperbolic groups and spaces, their numerous definitions and properties, examples, the ideal boundary, quasiconformal and conformal structures on the ideal boundary, cubical complexes, ...

No textbook is required. The following sources might be helpful, among many other.

- Magnus, Karras, Solitar. Combinatorial group theory.
- Lyndon, Schupp. Combinatorial group theory.
- Jean-Pierre Serre. Trees.
- Ghys, Haefliger, Verjovsky. Group theory from a geometrical viewpoint.
- Collins, Grigorchuk, Kurchanov, Zieschang. Combinatorial group theory and applications to geometry.
- John Meier. Groups, graphs and trees.
- Gilbert Baumslag. Topics in combinatorial group theory.
- Bridson, Haefliger. Metric spaces of non-positive curvature.