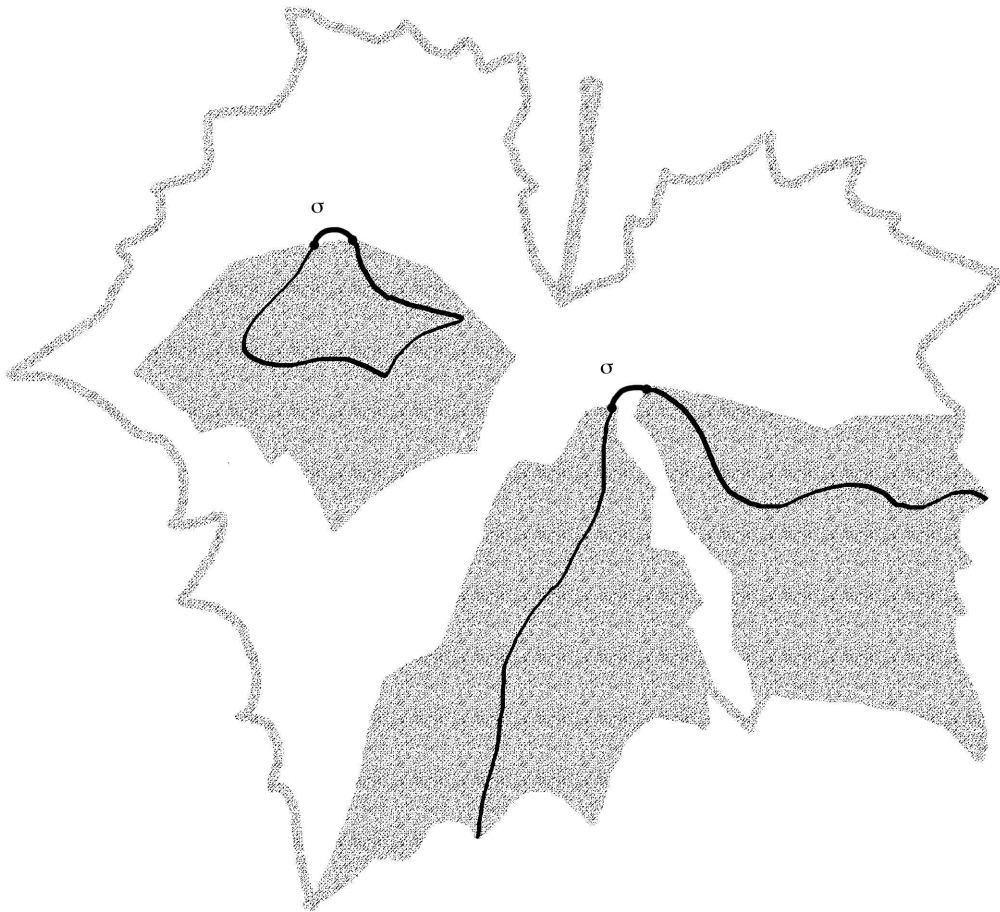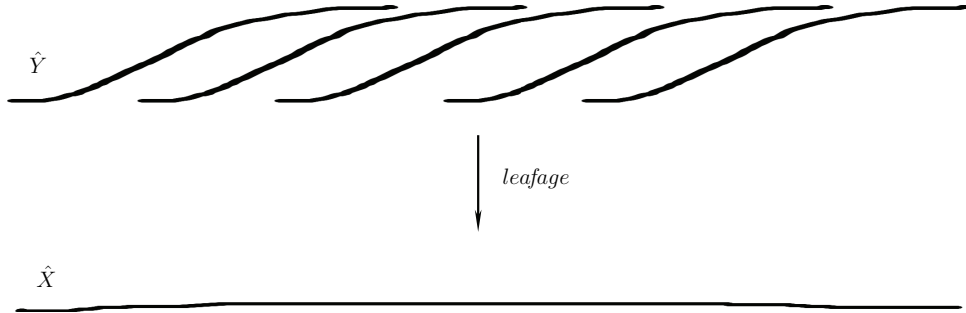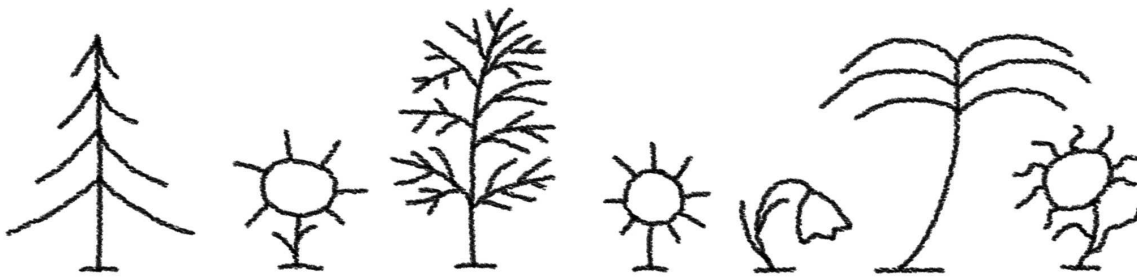# GROUPS, GRAPHS, AND
THE HANNA NEUMANN CONJECTURE

IGOR MINEYEV

ABSTRACT. Submultiplicativity, an analytic property generalizing the Strengthened Hanna Neumann Conjecture (SHNC) to complexes was proved in [1] assuming the deep-fall property. This implied SHNC. The purpose of this note is to present a proof of the original SHNC purely in terms of groups and graphs. We also give explicit examples showing that the upper bound in SHNC is sharp.
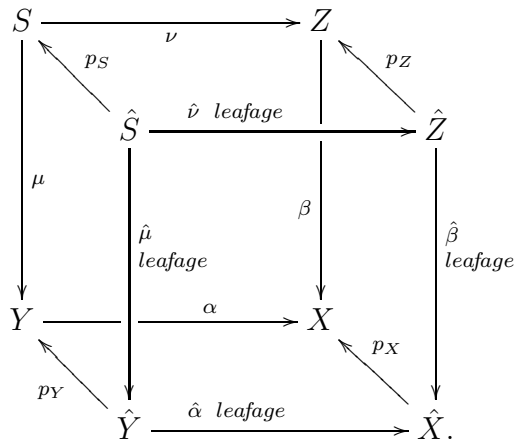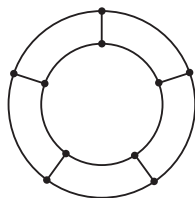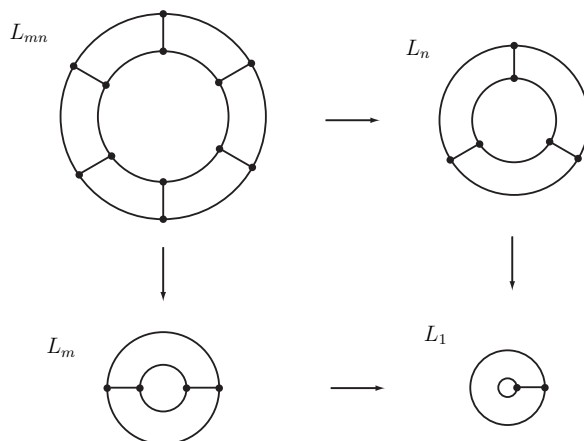
$$\hat{Y}$$

$$\downarrow \text{leafage}$$

$$\hat{X}$$

A garden.

A system of complexes.

$$
\begin{array}{ccc}
S & \xrightarrow{\ \nu\ } & Z \\
\ \nwarrow p_S & & \nwarrow p_Z \\
& \hat{S} \xrightarrow{\ \hat{\nu}\ \ leafage\ } \hat{Z} & \\
\mu \downarrow & \hat{\mu}\ \downarrow\ leafage \qquad \beta \downarrow & \downarrow \hat{\beta}\ leafage \\
Y & \xrightarrow{\ \alpha\ } & X \\
\ \nwarrow p_Y & & \nwarrow p_X \\
& \hat{Y} \xrightarrow{\ \hat{\alpha}\ \ leafage\ } \hat{X}. &
\end{array}
$$

The ladder L_m.

$$\bar{r}(L_m) = m$$

$$\bar{r}(L_{mn}) = \bar{r}(L_m) \cdot \bar{r}(L_n)$$

REFERENCES

[1] I. MINEYEV, *Submultiplicativity and the Hanna Neumann Conjecture.*, Ann. of Math., 175 (2012), pp. 393–414.

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